Economic Design of X-bar Chart Using Genetic Algorithm

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Under the guidance of
Dr. S. K. Patel

Department of Mechanical Engineering
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Dedicated

to my

Parents, Guide and Friends

Department of Mechanical Engineering

NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

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CERTIFICATE

This is to certify that the report entitled, **“Economic Design of X-bar Chart Using Genetic Algorithm**” submitted by **Mr. Abhijit Roy,** Roll No. **212ME2299** in partial fulfillment of the requirements for the award of **Master of Technology** in **Mechanical Engineering** with **“Production Engineering”** Specialization during session 2013-2014 in the Department of Mechanical Engineering in National Institute of Technology Rourkela, is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in this report has not been submitted to any other University/Institute for award of any Degree or Diploma.

Date:

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ABSTRACT

Control chart is a key tool in Statistical Process Control. This chart is one type of statistical tool which is used to monitor the quality of a process. It gives a visual representation of the status of the process indication whether the process is under control or not. It is used for finding any variation present in any process. Control charts display the variation in a process, so that anyone can easily determine whether the process is within control or it is out of control. For the design of X-bar control chart we need to find the optimal values of sample size, sampling frequency and width of control limit. In our work, we made a computer program in MATLAB based on Genetic Algorithm for finding the optimal values of above three parameters so that the total expected cost is minimized. Our result showed that Genetic Algorithm provides better result as compared to others reported in the literature.

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List of Symbols

μ_0 = Process mean

σ = Standard deviation

k = Width of control limit

n = Sample size

α = Probability of occurrence of Type-I error

β = Probability of occurrence of Type- II error

$\Phi(z)$ = Standard normal density

δ = Shift parameter

$\boldsymbol{\lambda}$ = Rate of occurrence of assignable causes

τ = Expected time of occurrence

g = time in hour for measurement

D = Time for finding assignable causes

E(T) = Expected length of cycle

E(C) =Net income in a cycle

E(A) =Net income per hour

E(L) = Loss cost per hour

V_0 = Hourly income for in-control operation

V_1 = Hourly income for out-of-control operation

a_1 = a =Fixed cost

a_2 =b = Variable cost

a_3 = Cost for finding assignable cause

a_3'= Cost for finding false alarm

a_4 = cost of operating in out-of-control state for one hour

ARL_0 = Minimum value (lower bound) on the in-control

ARL_1 = Maximum value (upper bound) on the out-of-control

s = Expected number of samples while taken in control

Chapter 1

Introduction

1. INTRODUCTION

One very important factor for any business or manufacturers or service provider to understand is that nothing stays the same as time changes. With time a lot of factors change, quality changes, variation occur, a lot of factor come and go as process continues, and therefore we end up with some variation. But for a product if there is a lot of variation, the customer or user will not be satisfied with that. Hence for this purpose we need some sort of control device, which will inform us when there is too much variation, i.e. there will be some kind of feedback mechanism. It will look at the result of the output, compare with the desired result or nominal level of quality and if the deviation is too large it will trigger a control action. This is the basic principle of any type of control. In case of statistical process control what we do is we let this control be activated when the data shows an exceptional behavior, then we apply some sort of corrective measures or decisions to minimize these variations.

1.1. Statistical process control

It is a device which helps in monitoring the quality of a product. They help in monitoring the quality online, which means that as the output are coming out we measure some desired characteristics of the output. Then we collect some data as samples taken out of the output and then verify whether the process is in-control or out-of-control.

Statistical Process control is applied during production, so that we can be saved from the loss which may occur due to production of faulty product. Statistical Process Control is very powerful but also very simple collection of tools. They will tell us whether the variations are within the tolerance limit, or outside the tolerance limit. Thus tells us whether to leave the process as it is, or take some corrective measures. They help us to analyze data's graphically and tell us when we have a problem and when we need to solve them.

1.2. Types of variations

There are two types of variation. They are:

(i) Random variation

- They are common cause variation,
- they are generally inherent in the process, and
- their elimination is only possible by improvement in the system.

(ii) Non-Random variation

- They are special cause variation,
- they occur due to recognizable factors, and
- they can be modified either by management actions or by operator.

These non-random or special cause variations are the ones which we generally identify by control chart, and are the one which we need to minimize.

Common cause variations are present in large number but cause very small variation. They do not have very large impact on the process.

1.3. Causes for variation

Some of the causes of variation have been named below:

(i) The Machine**:**

- Inherent Precision,
- Set-up,
- Machine condition, etc.

(ii) The Material:

- Moisture content,
- Bending,
- Contamination, etc.

(iii) The Method:

- Cutting speeds,

- Temperature,
- Procedures, etc.

(iv) The Operator:

- Technique,
- Training,
- Supervision, etc.

(v) Management:

- Poor process management,
- Poor systems, etc.

1.4. Control chart

Control chart is a key tool in Statistical Process Control (SPC). Control chart is a type of statistical tool which is used to check the quality of a product. They make the behavior of the process visible to us. They are used for finding any variation present in any process. Control charts display the variation in a process, so that anyone can easily determine whether the process is within control or it is out of control.

All process in this world will have some variation. It is impossible to have any process without any variation. But there are two different causes for these variations. One type of variation occurs due to reasons which are normally present within the process, hence they are termed as common cause variation. Another type is the one which occurs as a result of some special cause, hence they are called as special cause variation. Normal cause variations are present in large number but do not have much impact since they cause very small variation. Special cause variation on the other hand have large impact on any process, hence we need to minimize it.

1.5. Parts of control chart

There are three different parts in a control chart:

(i) Centre line,

(ii) Upper control limit and

(iii) Lower control limit.

The Fig. 1.1 shows the different parts of a control chart,

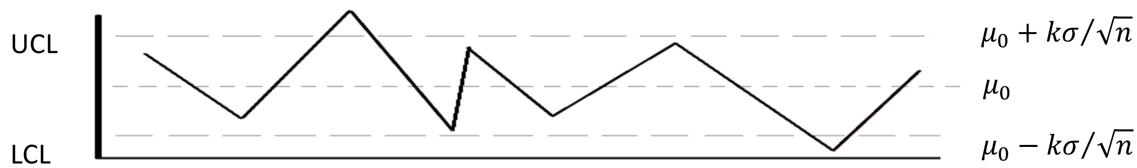

Fig. 1.1. X-bar Control Chart

Value of upper control limit and lower control limit depend on four factors. They are the process mean (μ_0), control limit width (k), standard deviation of the process (σ) and size of sample (n). When values are within the UCL and LCL then the process is said to be in control, but when points are above UCL or below LCL then it is said to be out of control.

Shewart [1] in his work suggested the 3-sigma control limit. Hence normally we generally set the UCL and LCL at ± 3σ from the mean of the process. Variation in control limit and percent of items captured is shown in the Fig. 1.2. Here we are assuming that it is a normally distributed curve.

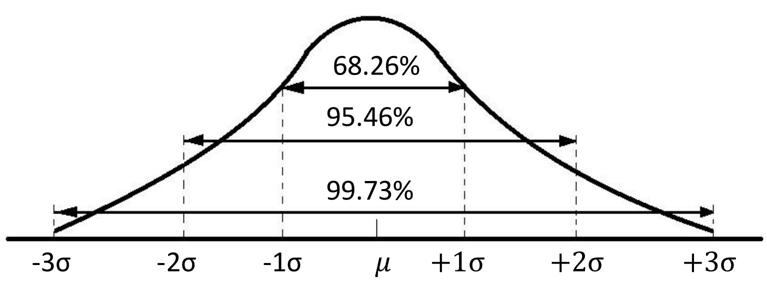

Fig. 1.2. Area under Normal Curve

X-bar control chart is a type of control chart. It helps in detecting whether a process is within normal variation. They play a very vital role in any process, because they help in minimizing the loss incurred by manufacturing defective products. They detect whether a process is out of control or in other words manufacturing defective items, hence necessary actions can be taken to prevent losses. For example, Crossley [2] took a shaft production process, in this main objective was to produce shaft of correct diameter. If shaft of wrong diameters are produced, this means the company will have to face heavy loss due to dispute products. Here comes X-bar chart into play, they help in determining any shift in diameter of the shaft.

1.6. Types of control chart

Depending on the type of data control chart can be classified into two types. They are Variable and Attribute control chart.

1.6.1. Variable control chart

They are used for parameters which are continuous in nature and can be measured. They can be used for parameters like temperature, weight, distance etc. They can have both decimals and fraction values.

1.6.2. Attribute control chart

These charts are used to measure quality characteristics. These charts are used when we need to determine the presence or determine the absence of any cause. They can be used when we need to determine the acceptance or rejection, or success or failure etc.

Hence we can also say that there are two types of data.

- **Variables** – They consists of measurable quantities, like mass, temperature, etc.
- **Attributes** – Things which we cannot measure but we need to count, like percentage of faulty items in a lot, defect type, etc.

1.7. Control charts for variable data

1.7.1. X-bar chart

They are the mean chart. They are used for controlling the accuracy. In X-bar chart we take a few samples and then calculate the mean of samples. It is this mean values which we plot on the X-bar chart. For center line we calculate the mean of sample means, and known as $\bar{\bar{X}}$.

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{n} \bar{x}_i}{n} \tag{1.1}$$

$$UCL = \bar{\bar{x}} + 3\sigma \tag{1.2}$$

$$LCL = \bar{\bar{x}} - 3\sigma \tag{1.3}$$

And σ is the standard deviation.

1.7.2. R chart

R charts are also known as range chart. They are used for controlling the precision. They use the amount of deviation in a lot. If the range is large, it means that individual in a subgroup have a lot of variation.

$$R = \max(x_i) - \min(x_i) \tag{1.4}$$

$$\bar{R} = \frac{\sum_{i=1}^{n} R_i}{n} \tag{1.5}$$

$$UCL = D_4.\ \bar{R} \tag{1.6}$$

$$LCL = D_3.\ \bar{R} \tag{1.7}$$

Value of D_3 and D_4 can be found from the Table 1.1.

Table 1.1. Value of D_3 and D_4.

Sample size	D_3	D_4
2	0	3.27
3	0	2.57
4	0	2.28
5	0	2.11
6	0	2.00
7	0.08	1.92
8	0.14	1.86

1.7.3. s chart

Standard deviation of the samples is plotted in this chart for controlling the variability in a process.

1.7.4. s^2 chart

In this chart the sample variance are plotted for controlling the variability of a process.

1.8. Control charts for attribute data

1.8.1. p chart

p charts are the type of control charts which are used to monitor the proportion of defective items in a sample. The control limits in these charts are based on binomial distribution.

1.8.2. c chart

c chart plots the number of defective items in the sample. The control limits in these charts are based on Poisson distribution.

1.8.3. u chart

u charts are similar to c chart, but only differs in the sense that unequal size samples can also be used here. Since sample size differs, hence control limit will also change accordingly.

1.9. Design of control chart

The three important parameters of a control charts are sample size, sampling frequency and width of control limits. The determination of these three parameters is known as the design of control chart. Design of control chart is of three types:

(i) Statistical design of control chart,

(ii) Economic design of control chart, and

(iii) Statistical economic design of control chart.

1.9.1. Statistical design of control chart

There are two types of errors which are always incorporated in the control chart. They will be always present since we do not carry 100% inspection. These two statistical errors are known as Type-I and Type-II error. In this design we need to minimize these two errors.

(a) Type-I error: This type of errors occurs when the control chart tells that the process is out-of-control but actually it is in-control. It is similar to having a false alarm. Here α shows the probability of Type-I error and has been shown in Fig. 1.3.

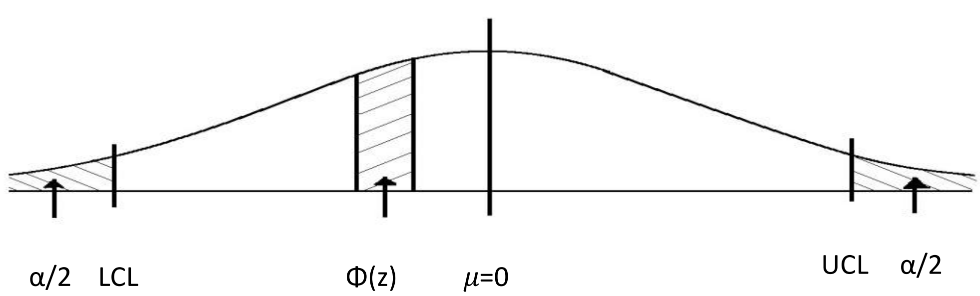

Fig. 1.3. Type-I error

And the probability of occurrence of Type-I error is given in Equation 1.8.

$$\propto = 2 \int_{k}^{\infty} \varphi\,(z)dz \tag{1.8}$$

(b) Type-II error: Type II error occurs when the control chart says that the process is in control but actually it is out of control. Probability of occurrence of Type-II error is denoted by β and has been shown in Fig. 1.4. Its power is given by Equation 1.9.

$$1 - \beta = \int_{\infty}^{-k-\delta\sqrt{n}} \varphi(z)dz + \int_{k-\delta\sqrt{n}}^{\infty} \varphi(z)dz \tag{1.9}$$

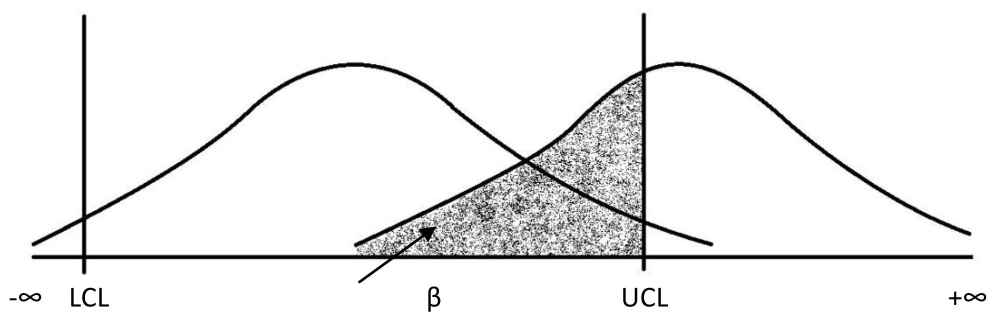

Fig. 1.4. Type-II error

1.9.2. Economic design of control chart

In this design our main objective is to reduce the total cost. For minimizing the total economic cost we need to select the optimal values of sample size, sampling interval and width of control limit and this is known as the economic design of control chart.

1.9.3. Statistical economic design of control chart

In this design we try to minimize the total cost as well as try to minimize Type-I and Type-II errors. This design is the integration of the above two design.

Chapter 2

Literature Review

2. LITERATURE REVIEW

For the understanding of our topic and knowledge about related works we have gone through this research papers.

Weiler [3] in his paper derived a formula by which sample size which is most suitable can be found out. This will help in finding out any variation in population mean and will require very less number of inspections for detection. His aim was to obtain sample size which is the most economical. He found out that the determination of sample size depends on the value by which the mean of population has shifted. He showed that if the mean of a population changes from μ to μ+kσ, where value of σ is fixed, then sample size 'n' which will be most economical will depend on only one factor k.

Chen et al. [4] stated that preventive maintenance helps in reducing the rate of failure by a quantity which is proportional to the level of preventive maintenance. In his paper he made a model by integrating economic design of x-bar control chart and preventive maintenance. They based on economic point of view proposed a moving average control chart with Weibull failure mechanism. They stated that with increase in the system running time it is better to reduce sampling interval. Their proposed chart is useful in monitoring the products obtained in continuous process or monitoring the quality of raw materials. In their research they studied the effect of variable sampling length instead of fixed sampling length. They formulated the loss cost function and they used a BASIC program for finding the optimal value of design parameters. Finally he performed some numerical calculation to understand the effectiveness of the model.

Cai et al. [5] studied about the cost involved in monitoring and adjustment of various processes. Processes need periodic adjustment having deteriorating properties. When statistical acceptance control charts are used for monitoring purpose the main determining factor is adjustment interval length. By using the economic design this parameter can be optimized such that we can achieve highest production economy within acceptable limit of quality.

Thilakarathne and Daundasekera [6] found out the maximum value of average income for a process by determining the optimal value of sample size, width of control limit and sample frequency. Finally they presented the output of the economic model in a tabulated pattern which helps the quality controller to determine their required parameters according to α and β value, for achieving maximum profit. They showed that with increase in sample size there is a decrease in expected income.

Liu et al. [7] stated that we use a control chart for monitoring a production process then we need to determine three parameters. These three parameters are the sample size, the sampling frequency and the width of the control limit. In their paper they developed a minimum loss design for a control chart, and for this purpose they took the loss function from Taguchi's model. For this purpose they took an example of a process, and this process was an orange juice production. They found out that when measurements are positively correlated for samples, it results in sample of smaller size and frequent sampling interval. They also found out that when correlation coefficient value is increased from 0 to 0.9, chart's power worsens. They also found that when measurement are negatively correlated for samples, it results in sample of smaller size and smaller width of control limit, but it was found that data which are negatively correlated don't have much significant effect on sampling frequency.

Chen et al. [8] found that x-bar control chart which have a variable sampling interval as well as sample size (VSSI) are better than normal x bar control chart which have fixed sampling interval and sampling size. But for situation when there is a need for prevention of production of defective items, the VSSI charts are costly. So in this paper they developed an evolutionary method by which we can find the optimal value of sample interval, sample size and width of control limit and the warning limit. They developed a model by integrating the Yang's correlation model and Costa's cost model and monitored the mean of a sample for correlated process values. They solved an industrial example and took the help of genetic algorithm for finding the optimal values. For finding the effect of input parameters on the economic design they then carried out sensitivity analysis. They found that value of shifting of mean has great effect on the smallest and largest sampling intervals, and also on the control limit. Hence for economic and effective design of VSSI charts the value for shift of mean should be carefully calculated.

Bashiri et al. [9] in this paper aimed at optimizing not only the cost function but also other parameters like Type-I error probability, average time to signal etc. For this purpose they proposed a multi-objective Genetic Algorithm. Then they compared their result with other numerical problems and showed the effectiveness of their design. They also performed sensitivity analysis of their design to check its robustness. They found that if we consider ATS along with other objectives it improves the robustness of the design, while other factor may degrade a little.

Kaya [10] stated that one important problem in design of control chart, is the determination of sample size. In this paper Kaya made a new approach for determining the sample size for attribute control chart. He took an example for the manufacture of piston based on probability of maximum acceptance and minimum cost and determined the optimal sample size by applying genetic algorithm. They also suggested a different structure of chromosome for increasing the efficiency of genetic algorithm. They performed five different type of cross-over and mutation and compared their result, because the performance and efficiency of genetic algorithm depends on cross-over and mutation. Finally they found that genetic algorithm was able to find better solutions.

Trong et al. [11] took the help of genetic algorithm for economically designing double sampling X-bar control chart (DS). This type of chart can very quickly detect any shift in process mean and are also able to reduce the size of sample effectively. However in real life situations parameters are interrelated and hence DS chart will result in high cost if their detection is wrong. In this paper they developed an economic design for DS control chart and determined the optimal values of sampling interval, sample size and control limit width and warning limits. They used genetic algorithm to solve an ice packaging process and find the optimal values. On the basis of sensitivity analysis they found out that when high positive correlation is indicated by samples, DS control chart provides greater expected cost while controlling shift of smaller size. They also found that total cost can be reduced effectively if occurrence of assignable cause, examining, sampling time can be reduced.

Celano and Fichera [12] stated that most important requirement of total quality management is to prevent the production of defective items. For this purpose we take the help of control chart for detection in any shift of the process mean, but doing so we increase the production cost. In this paper they proposed a new approach for solving this problem. This approach was based on evolutionary algorithm, they proposed a multi-objective approach and compared their result with other available heuristics approach. They result showed the superiority of their model over others.

Vommi and Seetala [13] made a new approach to develop a robust control chart. They said that designing of a control chart simply means to find the optimal value of parameters for which the control chart operates efficiently. For economic design the main aim is to choose the value of parameters such that total cost of the process is minimized. For a x-bar control chart this three parameters are sample size, sampling interval and the width of control chart. There are also various input parameters, like 'process failure rates' and 'cost of false alarm'. For an effective design of control chart this parameters needs to be accurately estimated. In case of conventional control chart they take point estimates, and hence in many case there are a lot of variation from its actual values. Hence heavy loss occurs due to this error. For reducing this loss this input parameters may be expressed in range, the actual value lying somewhere between this range. Subsequently there is also the need for choosing the best range for these parameters. In this paper they made an economic design of x-bar control chart and determined the optimal range for the input parameters. They used genetic algorithm for finding the optimal values.

Sumanta [14] in his paper has briefly told about genetic algorithm. There are various optimization techniques by which we can optimize a function, but in case of a multimodal function one problem which most optimization technique face is robustness, which is not the case with genetic algorithm. In this paper he solved some examples using genetic algorithm in MATLAB and gave detailed explanation. He said that although for unimodal function a lot of other techniques are available which work faster and more efficiently, but for multimodal function they fails. Genetic algorithm although slow but is a robust technique and surely provides the optimal solution for the problem.

Montgomery [15] in his book has told about the various modern statistical methods used for quality control. This book provides thorough concepts and basis for applying under a variety of situation.

Duncan [16] was the first to use proper optimization techniques for finding the control chart parameters. His paper was the first paper which deals with fully economic model of a Shewart-type control chart. He assumed that when the process is in control it is represented by μ_0, and when an assignable cause occurs having magnitude δ, it changes the mean to either $\mu_0 + \delta\sigma$ or $\mu_0 - \delta\sigma$.

Deventer and Manna [17] stated that optimizing techniques used for finding the optimal value of control chart parameters are complicated and costly. They developed excel program for finding the control chart parameters for both economic design and economic statistical design of control chart. Their program provided easy, user friendly and low cost ways as compared to other approach which are available only in expensive software packages.

Lorenzen and Vance [18] said that from the economic view point control chart parameters which are considered are the sample size, sampling interval and width of control limit. They in their paper have considered a general process model and derived the hourly cost function. They also discussed the numerical techniques to minimize the cost function and also performed sensitivity analysis.

Chapter 3

Mathematical Model

3. MATHEMATICAL MODEL

In this chapter we have explained the various mathematical models which we have studied have used in our project work.

3.1. Montgomery

Montgomery [15] optimized the Duncan [16] model. The design of X-bar control chart depends on three parameters, they are

(i) n (sample size),

(ii) h (interval of sampling) and

(iii) k (control limit width).

For optimizing the value of Cost function we need to find the optimal value of these three parameters.

Duncan considered μ_0 to be the initial process mean for the in-control process. However assignable causes occurs which results in the shift of the process from the mean. They either get shifted from μ_0 to $\mu_0 + \delta\sigma$ or $\mu_0 - \delta\sigma$,

Where;

σ is the standard deviation of the process and

δ is the shift parameter.

For finding the control limits of the control chart we need to add k times of the standard deviation to process mean or subtract k times the standard deviation from the process mean, i.e.

UCL= $\mu_0 + k\sigma/\sqrt{n}$ and

LCL= $\mu_0 - k\sigma/\sqrt{n}$, where k is control limit co-efficient.

The assignable causes which occur are assumed to be occurring at a rate of $\boldsymbol{\lambda}$ per hour and according to Poisson's ratio.

3.1.1. Production cycle

A production cycle has four different periods:

(i) Period when the process is in-control,

(ii) Period when the process is out-of-control,

(iii) Sampling time and interpreting time and

(iv) Time for finding assignable cause.

The in control period is given by $1/\lambda$. It is also assumed that for an assignable cause occurring between j^{th} and $(j+1)^{th}$ samples, the expected time of occurrence is given by Equation 3.1.

$$\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})} \tag{3.1}$$

The probability for detection of any assignable cause for a sample is given by Equation 3.2.

$$1-\beta = \int_{\infty}^{-k-\delta\sqrt{n}} \varphi(z)dz + \int_{k-\delta\sqrt{n}}^{\infty} \varphi(z)dz \tag{3.2}$$

Here β is the Type-II error. This is the type of error in which the process is actually out-of-control, but the control chart says that it is in-control and $\Phi(z)=(2\pi)^{-1/2}\exp(-z^2/2)$ is the standard normal density. Also the probability of Type-I error, i.e. the probability that the control chart will indicate out-of-control process but actually it is in-control is given by the Equation 3.3.

$$\propto = 2\int_{k}^{\infty} \varphi\,(z)dz \tag{3.3}$$

They showed that expected length of the out-of control period is given by h/(1-β)-τ. Sampling and interpretation time is given by a constant g, hence length for this portion of cycle is given by gn. Time for finding a assignable cause is given by D.

The equation for expected length for the cycle is shown in Equation 3.4.

$$E(T) = \frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + D \qquad (3.4)$$

Let V_0 be the income for an hour for in-control operation and V_1 be the income for an hour for an out-of-control operation. The cost spent when we take a sample of size n is given by (a_1+a_2*n), where a_1 is the fixed cost and a_2 is the variable cost of sampling. For finding an assignable cause the cost is a_3 and for a false alarm is a_3'.

The number of samples which are taken before an shift is

$$\propto \sum_{j=0}^{\infty} \int_{jh}^{(j+1)h} j e^{-\lambda t} dt = \frac{\propto e^{-\lambda t}}{1-e^{-\lambda t}} \qquad (3.5)$$

For a cycle the net income is given by the Equation3.6

$$E(C) = V_0 \frac{1}{\lambda} + V_1 \left(\frac{h}{1-\beta} - \tau + gn + D \right) - a_3 - \frac{a_3' e^{-\lambda h}}{1 - e^{-\lambda h}} - (a_1 + a_2 n) \frac{E(T)}{h} \qquad (3.6)$$

Hence the net income per hour can be found out by dividing E(C) with E(T). So we obtain

$$E(A) = \frac{E(C)}{E(T)}$$

Putting the value of E(C) and E(T) from Equation 3.6 and Equation 3.4 respectively we get

$$E(A) = \frac{V_0(1/\lambda)+V_1[h/(1-\beta)-\tau+gn+D]-a_3-a_3'\propto e^{-\lambda h}/(1-e^{-\lambda h})}{1/\lambda+h/(1-\beta)-\tau+gn+D}\frac{a_1+a_2n}{h} \quad (3.7)$$

If a_4 be the penalty cost per hour for production in out-of-control state. Let $a_4=V_0-V_1$, then E(A) can be also written as:

$$E(A) = V_0 - \frac{(a_1+a_2n)}{h} - \frac{a_4[h/(1-\beta)-\tau+gn+D]-a_3-a_3'\propto e^{-\lambda h}/(1-e^{-\lambda h})}{1/\lambda+h/(1-\beta)-\tau+gn+D} - \frac{a_1+a_2n}{h}$$

$E(A)=V_0 - E(L)$

$$E(L) = \frac{(a_1+a_2n)}{h} + \frac{a_4[h/(1-\beta)-\tau+gn+D]-a_3-a_3'\propto e^{-\lambda h}/(1-e^{-\lambda h})}{1/\lambda+h/(1-\beta)-\tau+gn+D} \quad (3.8)$$

Thus we see that the loss cost per hour E(L) is a function of only three parameters n, h and k. Hence we need to optimize this three parameters for finding the optimal value for loss cost function.

3.2. Deveneter and Manna

Deventer and Manna [17] took the Lorenzen and Vance [18] model, they said that a process start in an in-control state and then shifts to an out-of-control state, the time between this is known a production time. They showed the expected cycle time to be

$$E(T) = \frac{1}{\theta} + (1-\gamma_1).s.\frac{T_0}{ARL_0} - \tau + ng + h(ARL_1) + T_1 + T_2 \quad (3.9)$$

The various costs involved per cycle are as follows:

(i) Cost involved by producing defective items,

(ii) Cost due to false alarm,

(iii) Cost involved in repairing assignable variation.

Hence total quality cost per cycle is given by Equation 3.10.

$$E(C) = \frac{C_0}{\Theta} + C_1(-\tau + ng + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2) + \frac{sY}{ARL_0} + W$$

$$+(a + bn)(\frac{\frac{1}{\Theta} - \tau + ng + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2}{h}) \tag{3.10}$$

Here the above function is dependent on only three variables n, h and k. We can get the total expected cost per unit time by dividing the total quality cost by the expected cycle time. It has been shown in Equation 3.11.

$$E(L) = \frac{E(C)}{E(T)}$$

$$E(L) = \frac{\frac{C_0}{\Theta} + C_1(-\tau + ng + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2) + \frac{sY}{ARL_0} + W}{\frac{1}{\Theta} + (1 - \gamma_1).s.\frac{T_0}{ARL_0} - \tau + ng + h(ARL_1) + T_1 + T_2}$$

$$+ \frac{(a+bn)(\frac{\frac{1}{\Theta} - \tau + ng + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2}{h})}{\frac{1}{\Theta} + (1 - \gamma_1).s.\frac{T_0}{ARL_0} - \tau + ng + h(ARL_1) + T_1 + T_2} \tag{3.11}$$

We have seen that the above equation depends on only three variable n, h and k. All other parameters W, Y, b, a, δ, Θ, C_1, C_0, g, T_1, T_0, T_2, γ_2 and γ_1 are fixed quantities.

There are two more parameters which we find, they are ARL_0 and ARL_1, but they are dependent on α and β. Since ARL_0 is equal to $1/\alpha$ and ARL_1 is equal to $1/(1-\beta)$. They in turn are dependent on n and k. Hence overall E(L) is a function of n, h and k, and we need to optimize the value of this three parameters for optimal value of E(L).

While the process is considered to be in an in-control state, the number of samples expected is given by

$$s = \sum_i^{\infty} iP \tag{3.12}$$

where iP the assignable cause which occurs between i^{th} and $(i+1)^{st}$ sample.

$$s = \frac{1}{e^{\Theta h} - 1} \tag{3.13}$$

The expected time of occurrence within this interval is given by the Equation 3.14.

$$\tau = \frac{1}{\Theta} - \frac{h}{e^{\Theta h} - 1} \tag{3.14}$$

Hence by optimizing n, h and k we will find the optimal value of E(L).

3.3. Objective of the present work

In our work we are going to use genetic algorithm for optimizing X-bar control chart. For this purpose we are going to solve some problems reported in literatures and check whether genetic algorithm can provide us better result.

Chapter 4

Genetic Algorithm

4. GENETIC ALGORITHM

Genetic Algorithm is a type of algorithm which mimics the operation of evolution. In this chapter we have provided a detailed explanation for the process of Genetic Algorithm.

4.1. Genetic algorithm

Genetic algorithm is a form of algorithm which helps in finding the optimal solution of a problem from a solution space. In this algorithm, initially a possible set of solutions are created which are referred as population. This population then evolves to find a better solution. The general form of the algorithm is given below:

1. First a population is created comprising of random solutions.
2. Then we have to repeat the following steps until termination criteria is met:
    (a) Random selection of two individual from population. More fit the individual more is its chance of selection.
    (b) Cross-over between the two to get a better one.
    (c) New individuals have a random chance to mutate. However this chance is very small, because we do not want the individuals to chance completely.
    (d) Replace old solutions with new one.
3. Finally the one with the highest fitness value is selected as the solution.

Population is a set of solutions. As the algorithm progresses new individuals replace the old ones, new are born and old die. In the population a single solution is termed as an individual. And how good solution the individual provides is termed as its fitness function. More fit an individual is, more is its chance of getting selected for cross-over. Two new individuals are produced by the cross-over of two old individuals. There is also some chance of mutation.

4.2. Some question and their asnswer

There are certain questions like;

1. How to represent an individual?

2. How to calculate the fitness of individual?

3. How to select individuals for breeding?

4. How to achieve cross-over?

5. How to achieve mutation of individual?

6. What should be the population size?

7. Termination criteria?

The answer of the above questions varies from problem to problem. But the last two questions can be discussed generally.

Population size can be anything. It can be small and also can be large. Larger the population size, more number of solutions will be available. Hence more number of variations in the population can be achieved by cross-over. This means that better solution can be obtained if the population size is large, rather than which can be achieved if the population size is small. Hence we should take the population size as large as possible. But larger the population size more time will be needed for the algorithm to run.

In the algorithm we saw the ending criteria are much undefined. It is because there are a lot of ways in which we can stop the algorithm. One way is to specify number of generation and many others. All the solution of the other question generally depends on the problem.

4.3. An illustrative example

We will try to find the solution to our above questions with the help of a simple example. Let's take a maximization problem

$$f\,(x, a, b, c) = x^3 + a^2 - b^2 - c^2 + 2bc - 3xa + xc - ab + 2 \tag{4.1}$$

Here it is clear that we need to find the optimal value's of x, a, b and c for finding the maximum value of the function. We will try to understand the steps involved in genetic algorithm while trying to solve this problem. This will help us in understanding better about the various steps involved.

4.3.1. How to represent an individual?

One of the simplest processes to do it is to have an array of four values. But larger the individual more number of variations can be achieved, hence better solution can be obtained. Researchers like Beasley[19] and Holland[20] in their work showed that when we represent individuals by bit strings, best result can be obtained. Let's see some values and understand how we can represent them. We will simply take bits for each variable and finally add all the four values together and get a single bit string.

Let x = 12,

a = 5,

b = 8 and

c = 11.

Then we represent it as:

1100 0101 1000 1011

4.3.2. How to calculate the fitness of individual?

Now we know how to represent an individual. Now our aim is to calculate the fitness of individual. In this we need to know about two terms, 'evaluation' and 'fitness' functions. The basic difference between the two is that, evaluation is an absolute quantity and fitness is a relative quantity. Fitness tells us how an individual is better than rest.

For our case we can calculate the value of *'f'* which will serve as our evaluation function. Let our population is shown by,

0001 0110 1000 0000

0111 0110 1110 1011

0110 1001 1111 0110

1010 1110 1000 0011

The values are given in the Table 4.1.

Table 4.1. Values for Population

Individual	x	a	b	c	*F*
0001011010000000	1	6	8	0	-91
0111011011101011	7	6	14	11	239
0110100111110110	6	9	15	6	-43
1010111010000011	10	14	8	3	671

To calculate fitness function there are many ways. We can use ordinal ranking method, in which individuals are listed according to their value of fitness function. The best solution having the highest rank and so on. We can also use averaging method. In averaging method we divide the evaluation values with the average evaluation value. For calculating the average value we added 100 to all the evaluation value. The ordinal and averaging values are given in Table 4.2.

Table 4.2. Fitness Function

Individual	evaluation	Ordinal	averaging
0001011010000000	-91	1	0.03
0111011011101011	239	3	0.81
0110100111110110	-43	2	0.19
1010111010000011	671	4	2.62

4.3.3. How to select individuals for breeding?

Normally individuals with higher fitness function have higher probability of getting selected. However there is no hard and fast rule for selection.

One way is to use the ordinal method. Which give more chance to individuals with more fitness function. In our example the fourth individual will get a chance of 40% for selection, the second individual will get a chance of 30% for selection. Similarly the third and first will get a chance of 20% and 10% for selection respectively. Another process can be by using average fitness value.

There are various others ways of selection also, like roulette wheel selection or rank selection methods.

(a)Roulette wheel selection

In roulette wheel selection individuals are selected according to their fitness value. More the value of their fitness, more area will they cover in the roulette wheel as shown in Fig. 4.1. and hence will have more chance of getting selected.

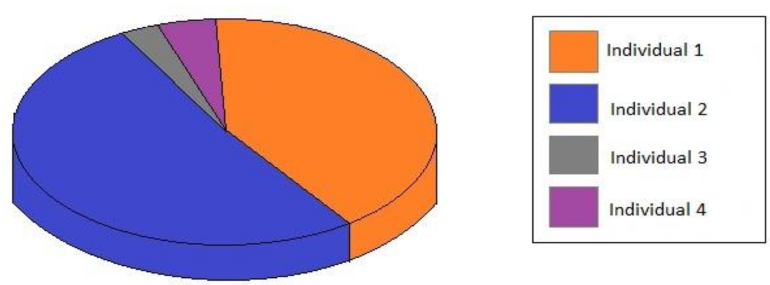

Fig. 4.1. Roulette wheel

But it has some problem, as in this case we can see that individual 1 and 2 will have maximum chance of getting selected, whereas individual 3 and 4 may not get selected at all.

(b) Rank selection

The above problem can be solved using rank selection method. In this first the rank of individual are decided, and then individual are assigned a new fitness value. The worst individual will have fitness value 1, the second worst will have fitness value 2 and so on. In our above case individual 3 will have fitness value 1, and individual 4, 1 and 2 will have fitness value 2, 3 and 4 respectively. This situation has been shown in Fig. 4.2 and Fig. 4.3 respectively.

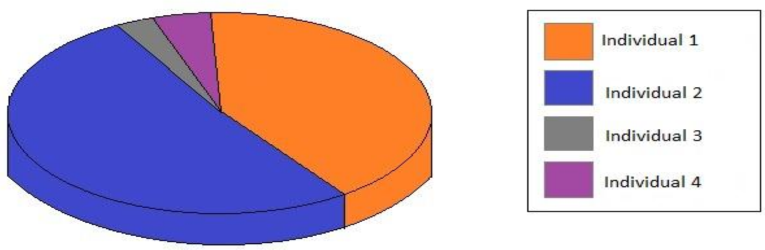

Fig. 4.2. Situation before ranking (graph of fitness)

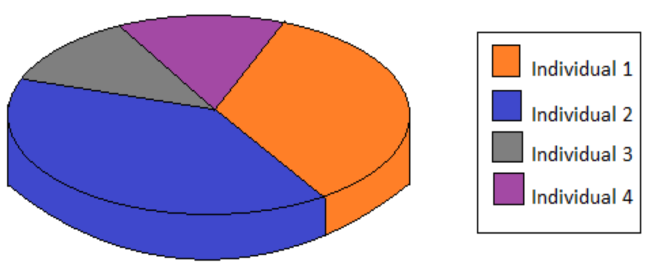

Fig. 4.3. Situation after ranking (graph of order numbers)

Now all the individuals will have a fair chance of selection. But the problem with it is that it will lead to slower convergence.

4.3.4. How to achieve cross-over?

Once our selection is complete, i.e. we have selected individuals, they are then crossed-over or breeding is done. Two new individuals are created from parents. There are many ways in which cross-over can be performed. Within the individuals, two locations are randomly chosen. This location refers to substrings. Then swapping of substrings is performed between them, and thus two individuals are created. Let's take the previous four individuals again:

0001 0110 1000 0000

0111 0110 1110 1011

0110 1001 1111 0110

1010 1110 1000 0011

Suppose the second individual and the fourth individual has been selected for cross-over. This also goes with the fact that they have the highest fitness function. However we should not forget that selection is a complete random event. The fourth to fourteenth bits have been selected. Then there is a swapping between the two individuals.

0111 0110 1110 1011	011***1 0110 1110 10***11	0110 1110 1000 0011
1010 1110 1000 0011	101***0 1110 1000 00***11	1011 0110 1110 1011

We should go on doing cross- over until the whole population is replaced with new individual. In our case there is a need for another cross-over. Now let's take that first and fourth individual have been selected randomly. One thing we should understand that, any individual can get selected more than one time, while some may not be selected for a single time. All this process is completely random. Let's perform one more cross-over:

0001 0110 1000 0000	0001 0110 1000 0000	0001 0110 1000 0011
1010 1110 1000 0011	1010 1110 1000 0011	1010 1110 1000 0000

So, now our new population is:

0110 1110 1000 0011

1011 0110 1110 1011

0001 0110 1000 0011

1010 1110 1000 0000

4.3.5. How to achieve mutation?

Now there also chance for some mutation. Mutation is very small, because we do not want to change the individual drastically, we want only small change. For achieving mutation some random flipping of bit is done, i.e. somewhere 0 is changed to 1, and somewhere 1 is changed to 0.

0110 1***1***10 1000 0011 → 0110 1***0***10 1000 0011

1011 0110 1110 1011 → 1011 0110 1110 1011

0***0***01 0110 1000 00***1***1 → 0***1***01 0110 1000 00***0***1

1010 1110 100***0*** 0000 → 1010 1110 100***1*** 0000

Here in our example the bits chosen for mutation are shown by bold and italics.

4.3.6. Final Calculation

In the final step let's calculate the above function. Results have been shown in Table 4.3.

Table 4.3. Final Values

Individual	x	a	b	c	*f*
0110 1010 1000 0011	6	10	8	3	51
1011 0110 1110 1011	11	6	14	11	1045
0101 0110 1000 0001	5	6	8	1	-19
1010 1110 1001 0000	10	14	9	0	571

Here the average value is found to be 412, which is higher than the average value of previous generation which was 194. Although this is a customized example, but this type of optimization is actually possible by genetic algorithm.

Chapter 5

Methodology

5. METHODOLOGY, RESULTS AND DISCUSSION

In our present work we have done the economic design of control chart using genetic algorithm. For this purpose we have taken an example which has been already been solved by Montgomery [15], and compared our result with that of theirs. In next part of our work we compared our result with that obtained by Deventer and Manna [17].

5.1. Numerical example-1

Here we have taken an numerical example from Montgomery [15]. Glass bottles are to be made, thickness of wall is an important criterion in this purpose. If it's very thin, then bottle will burst due to the internal pressure. For reducing the loss cost, the company wants an economic design of X-bar chart.

Various known parameters in the process are as follows:

a_1 = \$1, a_2 = \$0.10, a_3 = \$25, a_3' = \$50, a_4 = \$100, $\boldsymbol{\lambda}$ = 0.05, δ = 2.0, g = 0.0167 and D = 1.0 where:

a_1 is the fixed cost,

a_2 is the variable cost,

a_3 is the cost of investigating an action signal,

a_3' is the cost of investigating a false alarm,

a_4 is the cost of operating in out-of-control state for one hour,

$\boldsymbol{\lambda}$ is the mean frequency of process shift,

δ is the size of shift,

g is the time in hour for measurement, and

D is the average time to investigate any out of control signal.

For our work we have developed a MATLAB program based on genetic algorithm to best solution.

5.1.1. Result and discussion for numerical example-1

By our program we have calculated the optimum value of n, h and k and also calculated the minimum value of cost function. For this purpose we took range of n from 1 to 15, taking only integer value, h from 0.1 to 1 and k from 0.1 to 5 respectively. The result obtained is shown in Table 5.1. By observing the result we found that minimum value of cost function is obtained for n=5 and the corresponding h and k values are 0.815 and 2.982 respectively. The minimum value of cost function found is10.3675. We also observed that the value we obtained by genetic algorithm is superior to that obtained by Montgomery [15].

Table 5.1. Optimum Design of X-bar control chart

Sample size, n	Optimum sampling interval, h	Optimum width of control limit, k	Optimum cost, E(L)
1	0.499	2.296	14.6581
2	0.618	2.512	11.8766
3	0.706	2.679	10.8827
4	0.769	2.834	10.4901
5	**0.815**	**2.982**	**10.3675**
6	0.852	3.125	10.3804
7	0.882	3.263	10.4656
8	0.913	3.399	10.5897
9	0.942	3.531	10.7344
10	0.967	3.656	10.8903
11	0.993	3.788	11.0512
12	1	3.908	11.2147
13	1	4.032	11.3808
14	1	4.152	11.5484
15	1	4.269	11.7168

The graph between sample size and optimal value of economic cost is shown in Fig. 5.1.

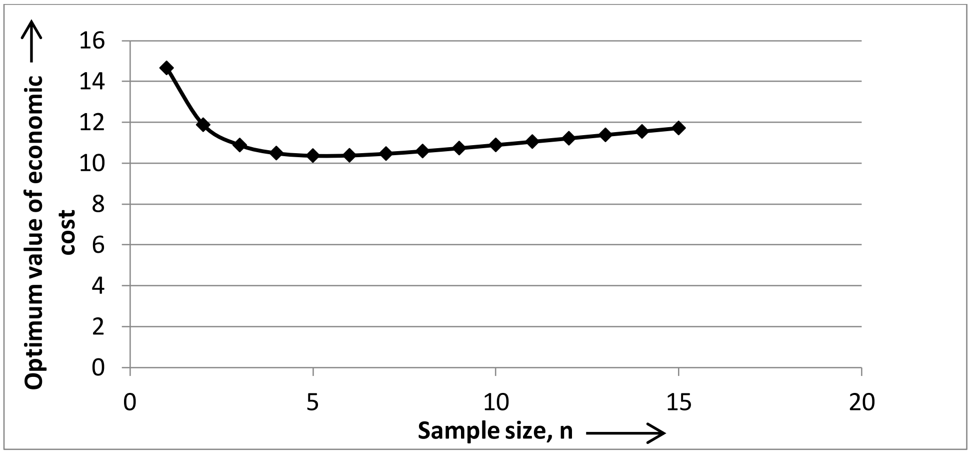

Fig.5.1. Optimum value versus Sample size.

Our result obtained was found to be superior to Montgomery [15]. There comparison is shown in the Table 5.2.

Table 5.2. Comparisons of Results

Sample size, n	Optimum cost, E(L)	
	Montgomery	Genetic Algorithm
1	14.71	14.6581
2	11.91	11.8766
3	10.90	10.8827
4	10.51	10.4901
5	**10.38**	**10.3675**
6	10.39	10.3804
7	10.48	10.4656
8	10.60	10.5897
9	10.75	10.7344
10	10.90	10.8903
11	11.06	11.0512
12	11.23	11.2147
13	11.39	11.3808
14	11.56	11.5484
15	11.72	11.7168

In the next part of our work, we also found out what is the effect of generation size on Economic cost. We found that on increasing number of generations the loss cost E(L) value decreases. The result we obtained is shown in Table 5.3.

Table 5.3. Variation in Optimum cost with increasing number of generation.

n	h	k	No of generations	EL)
1	0.624	2.184	10	14.7319
1	0.532	2.263	20	14.6642
1	0.512	2.268	30	14.6613
1	0.496	2.296	40	14.6581
1	0.499	2.296	50	14.6581
1	0.499	2.296	100	14.6581
1	0.499	2.296	200	14.6581
1	0.499	2.296	300	14.6581
1	0.499	2.296	400	14.6581
1	0.499	2.296	500	14.6581
n	**h**	**k**	**No of Generation**	**E(L)**
2	0.623	2.496	10	11.8786
2	0.618	2.513	20	11.8777
2	0.618	2.513	30	11.8777
2	0.618	2.512	40	11.8766
2	0.618	2.512	50	11.8766
2	0.618	2.512	100	11.8766
2	0.618	2.512	200	11.8766
2	0.618	2.512	300	11.8766
2	0.618	2.512	400	11.8766
2	0.618	2.512	500	11.8766
n	**h**	**k**	**No of Generation**	**E(L)**
3	0.776	2.648	10	10.8975
3	0.695	2.673	20	10.8835
3	0.706	2.678	30	10.8827

3	0.706	2.678	40	10.8827
3	0.706	2.679	50	10.8827
3	0.706	2.679	100	10.8827
3	0.706	2.679	200	10.8827
3	0.706	2.679	300	10.8827
3	0.706	2.679	400	10.8827
3	0.706	2.679	500	10.8827
n	**h**	**k**	**No of Generation**	**E(L)**
4	0.729	2.884	10	10.4962
4	0.768	2.838	20	10.4902
4	0.769	2.834	30	10.4901
4	0.769	2.834	40	10.4901
4	0.769	2.834	50	10.4901
4	0.769	2.834	100	10.4901
4	0.769	2.834	200	10.4901
4	0.769	2.834	300	10.4901
4	0.769	2.834	400	10.4901
4	0.769	2.834	500	10.4901
n	**h**	**k**	**No of Generation**	**E(L)**
5	0.817	2.993	10	10.3677
5	0.814	2.977	20	10.3676
5	0.815	2.982	30	10.3675
5	0.815	2.982	40	10.3675
5	0.815	2.982	50	10.3675
5	0.815	2.982	100	10.3675
5	0.815	2.982	200	10.3675
5	0.815	2.982	300	10.3675
5	0.815	2.982	400	10.3675
5	0.815	2.982	500	10.3675
n	**h**	**k**	**No of Generation**	**E(L)**
6	0.866	3.058	10	10.3842

6	0.852	3.130	20	10.3806
6	0.852	3.125	30	10.3804
6	0.852	3.125	40	10.3804
6	0.852	3.125	50	10.3804
6	0.852	3.125	100	10.3804
6	0.852	3.125	200	10.3804
6	0.852	3.125	300	10.3804
6	0.852	3.125	400	10.3804
6	0.852	3.125	500	10.3804
n	**h**	**k**	**No of Generation**	**E(L)**
7	0.868	3.353	10	10.4698
7	0.884	3.263	20	10.4657
7	0.882	3.263	30	10.4656
7	0.882	3.263	40	10.4656
7	0.882	3.263	50	10.4656
7	0.882	3.263	100	10.4656
7	0.882	3.263	200	10.4656
7	0.882	3.263	300	10.4656
7	0.882	3.263	400	10.4656
7	0.882	3.263	500	10.4656
n	**h**	**k**	**No of Generation**	**E(L)**
8	0.926	3.350	10	10.5908
8	0.906	3.401	20	10.5898
8	0.913	3.399	30	10.5897
8	0.913	3.399	40	10.5897
8	0.913	3.399	50	10.5897
8	0.913	3.399	100	10.5897
8	0.913	3.399	200	10.5897
8	0.913	3.399	300	10.5897
8	0.913	3.399	400	10.5897
8	0.913	3.399	500	10.5897

n	h	k	No of Generation	E(L)
9	0.916	3.541	10	10.7359
9	0.940	3.546	20	10.7346
9	0.942	3.531	30	10.7344
9	0.942	3.531	40	10.7344
9	0.942	3.531	50	10.7344
9	0.942	3.531	100	10.7344
9	0.942	3.531	200	10.7344
9	0.942	3.531	300	10.7344
9	0.942	3.531	400	10.7344
9	0.942	3.531	500	10.7344
n	h	k	No of Generation	E(L)
10	0.984	3.672	10	10.8911
10	0.965	3.645	20	10.8904
10	0.967	3.656	30	10.8903
10	0.967	3.656	40	10.8903
10	0.967	3.656	50	10.8903
10	0.967	3.656	100	10.8903
10	0.967	3.656	200	10.8903
10	0.967	3.656	300	10.8903
10	0.967	3.656	400	10.8903
10	0.967	3.656	500	10.8903
n	h	k	No of Generation	E(L)
11	0.999	3.831	10	11.0517
11	0.992	3.786	20	11.0513
11	0.993	3.788	30	11.0512
11	0.993	3.788	40	11.0512
11	0.993	3.788	50	11.0512
11	0.993	3.788	100	11.0512
11	0.993	3.788	200	11.0512
11	0.993	3.788	300	11.0512

11	0.993	3.788	400	11.0512
11	0.993	3.788	500	11.0512
n	h	k	No of Generation	E(L)
12	0.999	3.996	10	11.2154
12	1	3.906	20	11.2148
12	1	3.908	30	11.2147
12	1	3.908	40	11.2147
12	1	3.908	50	11.2147
12	1	3.908	100	11.2147
12	1	3.908	200	11.2147
12	1	3.908	300	11.2147
12	1	3.908	400	11.2147
12	1	3.908	500	11.2147
n	h	k	No of Generation	E(L)
13	0.979	3.823	10	11.3873
13	1	4.006	20	11.3809
13	1	4.032	30	11.3808
13	1	4.032	40	11.3808
13	1	4.032	50	11.3808
13	1	4.032	100	11.3808
13	1	4.032	200	11.3808
13	1	4.032	300	11.3808
13	1	4.032	400	11.3808
13	1	4.032	500	11.3808
n	h	k	No of Generation	E(L)
14	0.883	4.232	10	11.6171
14	0.989	4.155	20	11.5518
14	0.999	4.153	30	11.5487
14	1	4.152	40	11.5484
14	1	4.152	50	11.5484
14	1	4.152	100	11.5484

14	1	4.152	200	11.5484
14	1	4.152	300	11.5484
14	1	4.152	400	11.5484
14	1	4.152	500	11.5484
n	**h**	**k**	**No of Generation**	**E(L)**
15	0.946	4.496	10	11.7455
15	0.991	4.247	20	11.7204
15	0.999	4.268	30	11.7172
15	1	4.269	40	11.7168
15	1	4.269	50	11.7168
15	1	4.269	100	11.7168
15	1	4.269	200	11.7168
15	1	4.269	300	11.7168
15	1	4.269	400	11.7168
15	1	4.269	500	11.7168

Thus we see that with increase in the number of generation the cost function reduces.

5.2. Numerical example-2

Now going to the next part of our work we took an example from Deventer and Manna [17]. He in his work found out the optimal value of n, h and k for minimizing loss cost function. For this purpose also we developed a MATLAB program based on genetic algorithm and compared the results. As we already found that cost function reduces with increase in the number of generation, hence we omitted that work here. Since most of the X-bar chart in practice are based on 3-sigma limit, here in one part of our work we kept the width of control limit fixed, i.e. k=3, and found the value of cost function. In our next part we found the optimal value of n, h and k for calculating the optimal cost function.

Parameters provided are:

a = \$0.50, b = \$0.10, g = 0.05 hrs, $\delta = 1$, $\Theta = 0.01$, $T_1 = 2$, Y = \$50, W = \$25, C_0 = \$10/hr,

C_1 = \$100/hr, $T_0 = T_2 = 0$, $\gamma_1 = 1$, $\gamma_2 = 1$, $ARL_0 = 1/\alpha$, $ARL_1 = 1/(1-\beta)$; where

a is the fixed cost of sampling,

b is the variable cost,

g is the time in hour,

δ is the shift size,

Θ is the mean frequency of process shift,

T_1 is the time to investigate a action alarm,

Y is the cost to investigate a false alarm,

W is the cost to investigate a true signal,

C_0 is the hourly cost for operating in the in-control state,

C_1 is the hourly cost for operating in the out-of –control state,

T_0 and T_2 is the value that process continues during search and repair,

γ_1 and γ_2 is the indicator that the process continues during search and repair respectively,

ARL_0 is the minimum value (lower bound) on the in-control, and

ARL_1 is the minimum value (upper bound) on the out-of-control.

5.2.1. Result and discussion for numerical example-2

In the first part of our work we took value of k=3, since in actual practice x-bar chart are based on 3-sigma limit. We vary the value of n from 1 to 20, taking only integer value. The range of h was from 0.5 to 2.5. The result obtained is shown in Table 5.4. and relation between sample size and cost is shown in Fig. 5.2. The minimum value of cost function was found to be 14.9424, this value was obtained for n=14 and h=1.751 and k=3, since we fixed the value of k to 3.

Table 5.4. Optimum cost value for k=3

n	h	k	E(L)
1	0.5	3	28.8867
2	0.5	3	20.5808
3	0.5	3	17.8042
4	0.5	3	16.6638
5	0.588	3	16.1168
6	0.714	3	15.7518
7	0.844	3	15.4954
8	0.976	3	15.3095
9	1.217	3	15.1874
10	1.244	3	15.0806
11	1.376	3	15.0149
12	1.504	3	14.9720
13	1.630	3	14.9497
14	**1.751**	**3**	**14.9424**
15	1.866	3	14.9484
16	1.964	3	14.9656
17	2.077	3	14.9923
18	2.173	3	15.0271
19	2.263	3	15.0687
20	2.347	3	15.1162

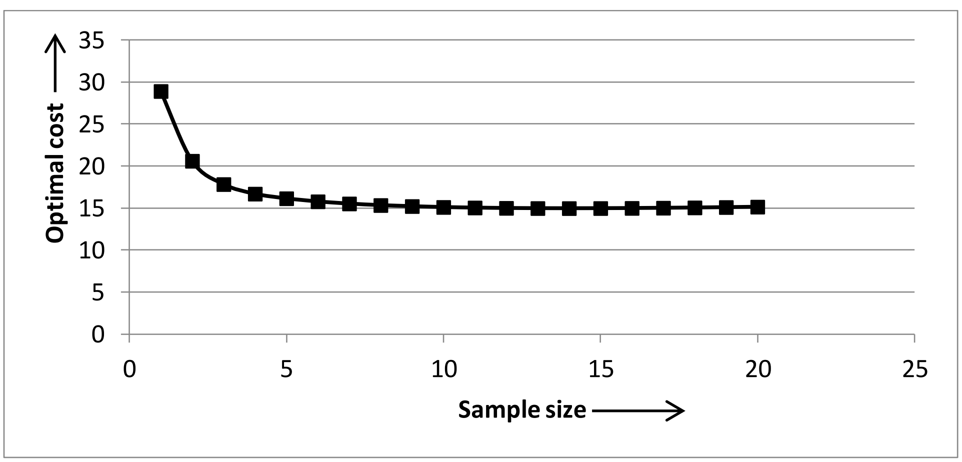

Fig. 5.2. Sample size versus Optimal cost

For the next part of our work we also found out the optimal value of k along with optimal value of n and h, for calculating the optimal value of E(L), i.e. cost function. For doing this we take the range of n from 1 to 20, h from 0.5 to 2.5 and k from 2 to 3. We found out that the minimum value of E(L) is 14.83 for n=12, h=1.843 and k=2.624. We also found out that the result obtained by us is superior to that obtained by Deventer and Manna [17]. Optimal value of cost function obtained by him was 14.90 while that obtained by us is 14.83. The comparison of results is shown in Table 5.5.

Table 5.5. Comparison of result.

n	Deventer and Manna			Genetic Algorithm		
	h	k	E(L)	h	k	E(L)
1	0.7	2.1	19.22	0.594	2.158	19.20
2	0.7	2.3	17.36	0.691	2.280	17.35
3	0.9	2.3	16.43	0.811	2.346	16.42
4	0.9	2.4	15.87	0.937	2.392	15.86
5	1.1	2.4	15.51	1.072	2.422	15.51
6	1.3	2.4	15.28	1.216	2.441	15.27
7	1.3	2.5	15.11	1.324	2.481	15.10
8	1.5	2.5	14.99	1.444	2.504	14.99
9	1.6	2.5	14.92	1.556	2.532	14.91
10	1.6	2.6	14.87	1.662	2.559	14.87
11	1.7	2.6	14.85	1.757	2.589	14.84
12	**1.9**	**2.6**	**14.84**	**1.843**	**2.624**	**14.83**
13	1.9	2.7	14.85	1.933	2.651	14.85
14	2.0	2.7	14.86	2.012	2.682	14.86
15	2.1	2.7	14.89	2.087	2.714	14.88
16	2.2	2.7	14.92	2.164	2.745	14.91
17	2.2	2.8	14.96	2.226	2.779	14.96
18	2.3	2.8	15.01	2.290	2.812	15.00
19	2.4	2.8	15.06	2.350	2.848	15.05
20	2.4	2.9	15.11	2.409	2.880	15.10

Chapter 6

Conclusions

6. CONCLUSIONS

In our work we have made the economic design of X-bar control chart and using Genetic Algorithm. Following are the conclusion which we arrived based on the result obtained.

- Genetic Algorithm provided superior result than that provided in the literature.
- For Montgomery problem minimum value of cost function was found to be 10.3675, and was obtained for n=5, h=0.815 and k=2.982.
- By increasing the number of generation the cost function reduces, i.e. we are able to find more optimal solution.
- For Deventer and Manna problem the minimum value of cost function was found to be 14.9424 and this value was obtained for n=14 and h=1.751 and k=3, since we fixed the value of k to 3.
- For the next part of our work we also optimized the value and k and found the minimum value of E(L) to be 14.83 for n=12, h=1.843 and k=2.624.

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